

# HALL MIDDLE SCHOOL MATH FESTIVAL DEC. 1, 2023

*Instructions:* Please visit several stations. Better to visit fewer stations and spend time there rather than rushing through all of the stations. When you have completed an activity, ask the facilitator to sign your sheet. Give this sheet to your teacher when you leave the festival. Have fun!

Your name:

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**1 Geometric Puzzles.** Try some easy tiling problems with pattern blocks and/or tangrams, and then explore harder challenges, including the 3-dimensional SOMA cube puzzle. Test your skills by asking a facilitator to time you when you assemble the giant cube!

Facilitator:

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**2 Games of Little or No chance.** Start with some easy pencil-and-paper games, like *Take-away*, and then graduate to the rigors of *Puppies and Kittens* and *Cat and Mouse*.

Facilitator:

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**3 Mathematical Magic for Muggles.** You will learn several interesting magic tricks that simulate superhuman abilities, but only require very simple mathematical tools.

Facilitator:

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**4 The Cautious Driver's Maze.** Mazes are not that hard, but this one has a tricky rule: you can only make right turns. Figure out how to complete the maze on paper, and when you are ready, ask the facilitator to time you on the larger maze on the floor.

Facilitator:

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*Please turn over for more activities*

**5 Mathematical Origami.** You will learn how to build beautiful complex polyhedra using simple folded subunits.

*Facilitator:*

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**6 The Rational Tangles Dance.** A mathematical “square dance” that requires four people. It is probably the most fun you can ever have with fractions!

*Facilitator:*

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**7 Minimal Surfaces with Soap Films.** When a shape is immersed in a soap solution, a thin film forms that attempts to minimize things like area, volume, or energy. Explore what happens with a variety of 2- and 3-dimensional objects.

*Facilitator:*

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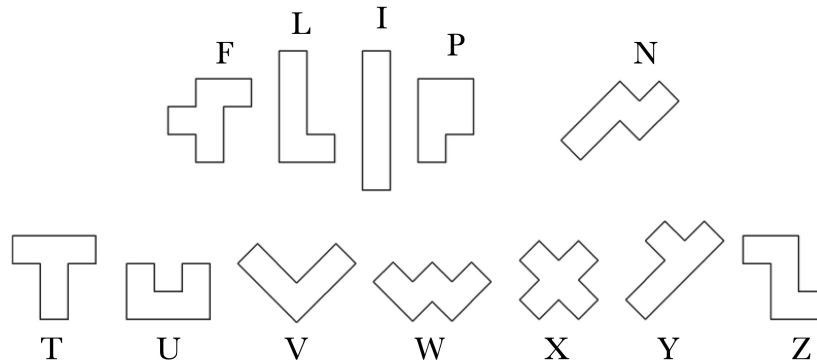
FOR MORE INFORMATION ABOUT THE HALL MIDDLE SCHOOL MATH FESTIVAL ACTIVITIES, VISIT [HTTPS://WWW.BAMO.ORG/HALLMS231201/](https://www.bamo.org/hallms231201/) OR SCAN THE QR CODE BELOW.



# 1 Geometric Puzzles

**Pentominos** The pentominos are the 12-different shapes made by gluing five squares together, edge to edge. These 12 shapes have convenient names, based on the letter of the alphabet that they resemble.

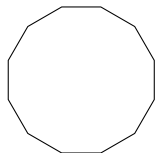
We have a collection of several complete sets of pentominos, each set in one of six colors.



- Since each pentomino has 5 squares, and there are 12 of them, can you tile a  $5 \times 12$  rectangle using one of each kind?
- What about smaller rectangles? And what if you can use the same pentomino more than once? Try  $5 \times 5$ ,  $3 \times 10$ ,  $5 \times 9$ , etc.
- The P pentomino is versatile. See what kind of rectangles you can tile using lots of Ps and only one other pentomino used once.

**Pattern blocks** These colorful blocks can be rearranged in astronomically many ways. For now, let's just concentrate on a few challenges.

- One of the blocks is a regular hexagon. Can you create a larger hexagon, whose side is twice the length of the original? Of course you can! How about three times as big?
- Can you make a regular dodecagon (12-sided polygon, all sides and angles equal)? Here is an example.



Can you make a dodecagon whose side length is the one unit (the side length of the hexagon)? How about two units? Three? Must your dodecagon be symmetrical?

**The SOMA cube** This puzzle was invented by a Danish scientist in the 1930s and simply consists of 7 three-dimensional polyhedra made by gluing cubes together, face to face. There are many fun challenges to try. Try them with the small wooden pieces, and then test your skills to reproduce them with the giant cardboard pieces. Ask a facilitator to time you!

- (a) Assemble a  $3 \times 3 \times 3$  cube, of course. It can be done in many different ways!  
 (b) Try to any of the following, using all the pieces.

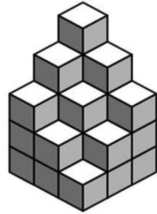
**THE CRYSTAL**

Image from Jan de Groot's Recreation of the 1967 Danish SOMA Booklet  
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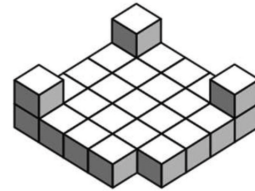
**THE CASTLE I**

Image from Jan de Groot's Recreation of the 1967 Danish SOMA Booklet  
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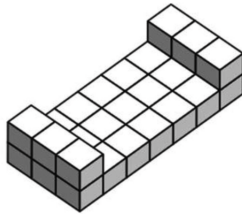
**THE BED**

Image from Jan de Groot's Recreation of the 1967 Danish SOMA Booklet  
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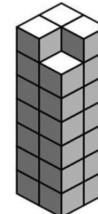
**THE TOWER**

Image from Jan de Groot's Recreation of the 1967 Danish SOMA Booklet  
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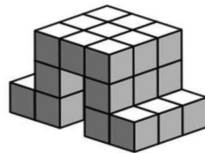
**THE TUNNEL**

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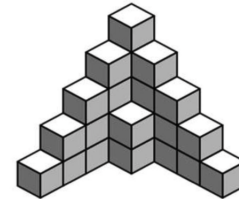
**THE CORNER STONE**

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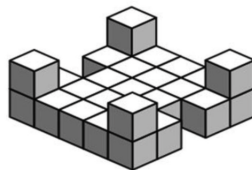
**THE CASTLE II**

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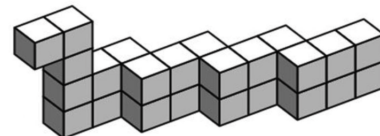
**THE SNAKE**

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## 2 Games of little to no chance

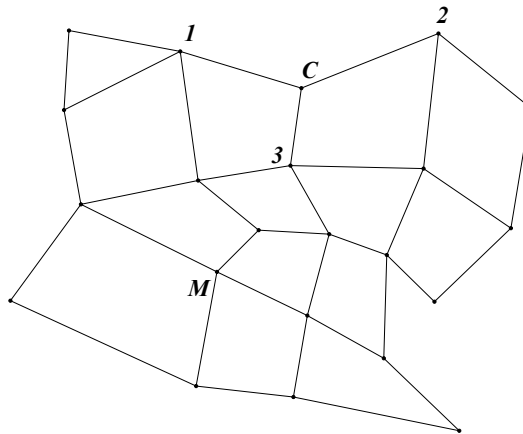
**Takeaway** This is a pencil-and-paper game played between two people,  $A$  and  $B$ , who alternate turns. Start with a number (up to you, but let's try 17). On each person's turn, the only legal move is to subtract 1, 2, 3, or 4. For example, player  $A$  subtracts 3, leaving  $B$  with 14 and  $B$  takes away 1, leaving  $A$  with 13, etc. The object of the game is to make the last legal move; i.e., to leave your opponent with zero (so they lose, since they cannot move). IS THERE A WINNING STRATEGY?

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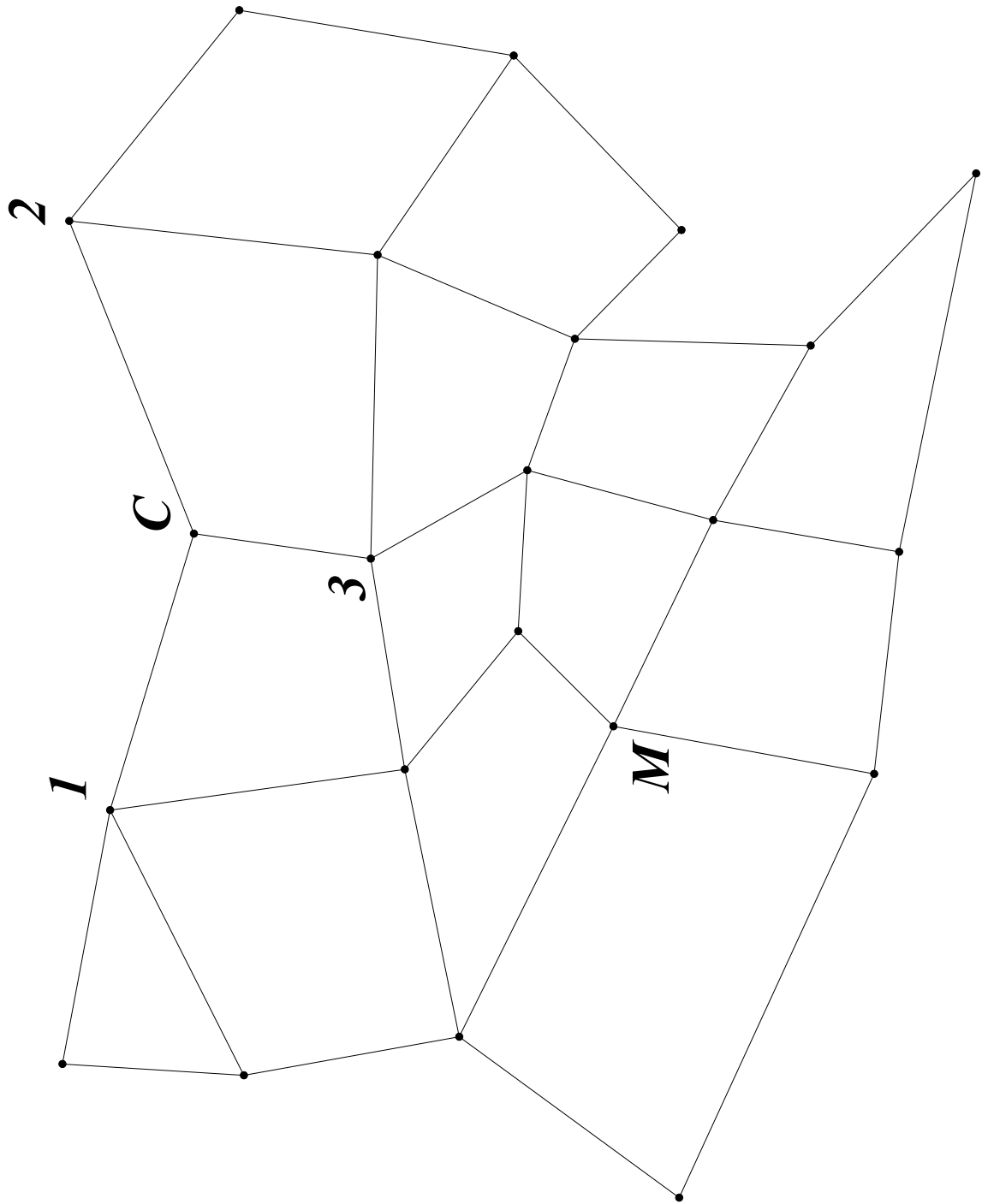
**Puppies and Kittens** This is also a two-person game pencil-and-paper game like before, and the object is to make the last legal move. You start with some puppies and kittens in a pet shelter; say 11 puppies and 14 kittens (but it is up to you). A legal move uses the two principles *have a heart* and *be fair*. The first principle means that when it is your turn, you *must* take away at least one animal! You can take as many as you want. The second principle means that *if you choose to take both puppies and kittens*, you must take equal numbers of both animal. For example,  $A$ 's first turn could be to take 4 puppies, or to take all 14 kittens, or to take 5 puppies and 5 kittens, but it would not be legal to take no animals, or to take, say, 4 puppies and 5 kittens. (Notice that taking all 14 kittens is a bad move, because then  $B$  wins by taking all the puppies.) IS THERE A WINNING STRATEGY?

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**Cat and Mouse** A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.



Initially, the cat is at the point labeled  $C$ ; the mouse is at  $M$ . The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. (Use the larger drawing on the other side to play.) WHICH ANIMAL DO YOU WANT TO BE? IS THERE A WINNING STRATEGY?



### 3 Mathematical Magic for Muggles

You will learn how to do up to three card tricks that simulate supernatural powers that are easy to perform and use very simple mathematical principles, such as  $x + -x = 0$ .

**Fingers that can see** The Magician deals cards on a table, placing them face up or face down on the command of the Participant, and stops dealing when the Participant says so.

Then the Magician is blindfolded. The Magician proceeds to put the cards into two piles, using their magical seeing fingers, so that, miraculously, each pile has exactly the same number of face-up cards!

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**Two random numbers** The Magician asks the Participant to choose a random number  $n$  between 1 and 20, and share this number with the audience without letting the Magician know. The Participant then removes the top  $n$  cards from the deck.

Next, the Magician deals 20 cards from the top of the diminished deck (which is missing  $n$  cards), and they ask the audience to notice the  $n$ th card dealt (without giving it away with body language!).

Next, an audience member is asked to say a random number that is smaller than the size of the now very diminished deck (it is missing  $20 + n$  cards). We call this number  $h$ . The Magician then deals  $h$  cards from the top, face-down. Then they place the stack of 20 cards on top of this, and put the rest of the diminished deck on top of that (so the  $n$  cards removed at the start are still missing).

Finally, the Magician deals cards off the top, but at some miraculous point, stops, and it is the one that the audience noted!

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**What is the name of this trick?** The Magician takes about half a deck and shows the cards in it to the Participant, who is invited to shuffle them. The magician then apparently messes the cards up further in a random way with respect to orientation (face-up vs. face-down). Then the Magician invites the Participant to continue messing up the cards with respect to orientation. Eventually the cards are in a single pile, but five of the cards have one orientation, and the rest have a different orientation. What is going on?

## 4 The Cautious Driver's Maze

Mazes are not that hard, but this one, which was first deployed at the National Museum of Mathematics (“MoMath”) in New York, has a tricky rule: *you can only make right turns*. Figure out how to complete the maze on paper, and when you are ready, ask the facilitator to time you on the larger maze on the floor.

